Power Efficient Topology Control for Wireless Networks with Switched Beam Directional Antennas

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***Abstract*— We consider the problem of power-efficient topology control with switched beam directional antennas taking into account their non-uniform radiation pattern within the beamwidth. Previous work have all assumed uniform gain for these antennas. We construct optimal algorithms that take into account a model of non-uniform gain. Moreover, we argue that with such a model of gain, antenna orientation is a significant parameter to be considered while constructing power-efficient topologies. We present heuristic algorithms that construct power-efficient topologies taking antenna orientation into consideration and demonstrate significant reductions in the power required to keep the network connected.**

I. Introduction

A widespread approach to topology control with omni-directional antennas is to find a power assignment for all nodes such that cost metrics like throughput or power consumption are optimized subject to constraints such as network connectivity [5], [7]. Such approaches to construct power-efficient topologies for wireless networks with omni-directional antennas have recently been adapted to the case of directional antennas [1], [2], [4], [10], [12]. Directional Antennas have received considerable attention the last few years due to the spatial-reuse they provide which increases the potential capacity of wireless ad-hoc networks [9], [13]. Moreover, there are also benefits of power efficiency with directional antennas as they radiate power in only the desired directions. The gains of these antennas are typically much higher than the omni-directional case making them influential in reducing the power required between a transmitter and receiver.

Two types of directional antennas have mainly been considered; steered beam and switched beam from the class of smart antennas. Steered beam directional antennas radiate power in only a certain direction, but the point of maximum gain can be steered as desired to always utilize the peak gain for transmitting and receiving. So the positioning of antennas, or antenna orientation, of nodes when deployed does not matter. However, these antennas require complex hardware and the cost makes them infeasible for most wireless ad-hoc networks. Switched beam directional antennas are less complex and cheaper, and thus, more feasible for ad-hoc wireless network settings in the coming few years [8]. However, the point of maximum gain cannot be steered by the communication system and is predefined based on the antenna orientation. Thus, if nodes are not communicating through the axis of maximum gain of each other, it could lead to considerably more power being required for that link.

Of the above mentioned previous work, the ones focusing on switched beam directional antennas ignore the effect of non-uniform gain and assume a constant uniform gain within the beamwidth. The circular disk model used for omni-directional antenna radiation pattern was simply cut into pie sections to model directional antennas. The reality is that the radiation pattern is far from uniform. We need to model the switched beam directional antenna radiation pattern with a radical approach that reflects the non-uniform gain within the beamwidth. For any algorithm, the challenge is to build topologies which use regions of higher gain as much as possible to reduce the power consumption. With uniform gain assumptions, the power required for communication between two nodes mainly depends only on the distance between them. But with a non-uniform radiation pattern, there is a new challenge of the gain in the desired direction also being a significant factor in deciding the required power.

In this paper, we address the issue of power-efficient topology control in a static wireless ad-hoc network setting with switched beam directional antennas, taking into account their non-uniform radiation pattern. We present a model which reflects the non-uniform radiation pattern of these antennas. We present optimal centralized algorithms that build topologies using this non-uniform model of gain. Moreover, when dealing with a non-uniform radiation pattern for switched beam antennas, antenna orientation needs to be considered. The positioning of a node's antenna decides what power each of its communication links incur. Thus, topology control algorithms need to find power assignments as well as antenna orientation for each node to optimize the power-based cost metric under consideration. We present heuristic algorithms which build topologies based on this approach and show that they are significantly more power-efficient.

The paper is structured as follows. Section II describes some basics about directional antennas followed by our propagation and antenna models. We formulate our problem in Section III describing our network setting, the power based cost metric and network property sought. In Section IV we present optimal algorithms for the case when the antenna orientation is given and is not a parameter for the topology construction. In Section V we present heuristic algorithms for our topology control problem with antenna orientation used as a parameter. Section VI gives the results of the evaluation of our algorithms. We finally conclude the paper in Section VII with directions for future research.

II. Directional Antennas

In this section we introduce some related terminology about directional antennas. We also present our models and related assumptions before we formulate the problem in the following section.

A. Gain and Beamwidth

An omni-directional antenna radiates or receives energy equally well in all directions while a directional antenna transmits/receives more energy in one direction compared to others. The term gain is used to quantify the directionality of the antenna. The gain of an omni-directional antenna is typically taken as unity[1].

Beamwidth is the angle subtended between the two points on either side of the direction of peak gain that are 3 dB down in gain. In terms of absolute values, a 3 dB drop in gain halves the peak gain. The more directional an antenna, smaller the beamwidth and higher the gain. However, due to presence of side lobes, two antennas with equal gain (beamwidth) may not have the same beamwidth (gain).

B. Propagation Model

The required transmit power for communication between a transmitter-receiver link is inversely proportional to the product of their gains and is specified as

$$P_t = \frac{c.d^\gamma}{G_t.G_r} \tag{1}$$

where P_t is the transmit power, d is the distance between sender and receiver, γ is the path loss exponent, c a constant depending on factors like antenna height and G_t and G_r being the gain of transmitting and receiving antenna towards each other. For Omni-Directional antennas, G_t and G_r are unity. Hence power required is directly proportional to d^γ. With directional antennas, however, the gains have a role to play in deciding power required for communication, as described subsequently.

C. Mode of Communication

The mode of communication with directional antennas can be classified into Directional-Omni (DO), or Directional-Directional (DD) based on the type of antenna used for transmission and reception across a communication link. DO links require more power than DD links as only the transmitter uses the higher gain providing directional antenna. However, coordination between the communicating nodes is easier. Due to omni-directional reception, now the sender can transmit packets directionally without worrying how the receiver's antenna is oriented. Omni-directional transmission with directional reception (OD) links are also an option, but suffer from requiring more power than DD links with no ease of coordination like DO links. Hence OD links are not considered in our work. This mode though is worth considering when the Effective Isotropic Radiated Power (EIRP) limits are a factor as is done in [11].

D. Switched Beam Antenna Model

We assume each node is capable of working in omni-directional mode too, either by adjusting the radiation pattern or having another omni-directional antenna. We do not consider varying the gain of the antennas for any other purpose and assume a constant beamwidth of θ for the directional mode. We use a simplifying assumption that the effects due to side-lobe interference are negligible. Furthermore we consider switched beam antennas with no nulls in between beams facing any direction. That is, there is $360°$ coverage (this also implies that there is some overlap among the beams which we ignore for simplicity as shown in Figure 1). By antenna orientation we mean the angle between the first beam anticlockwise from the positive x-axis and the x-axis. The orientation of antenna in Figure 1 is Φ. We will be concerned only with the azimuth plane for antenna orientation and radiation pattern in this work.

1) Bounds on required power due to non-uniform gain: Each beam's gain varies from a peak of G_{max} to $G_{max}/2$ at its borders[2]. Thus, regardless of direction of transmission or reception, transmitter and receiver nodes will have at least half the maximum gain towards each other.
Thus, for DD links we have

$$G_{max}/2 \leq G_t, G_r \leq G_{max} \tag{2}$$

or

$$\frac{{G_{max}}^2}{4} \leq G_t.G_r \leq {G_{max}}^2 \tag{3}$$

From Equation 1,

$$\frac{4.c.d^\gamma}{{G_{max}}^2} \geq P_t \geq \frac{c.d^\gamma}{{G_{max}}^2} \tag{4}$$

Similarly for DO links with G_r unity, we have

$$\frac{2.c.d^\gamma}{{G_{max}}^2} \geq P_t \geq \frac{c.d^\gamma}{{G_{max}}^2} \tag{5}$$

For simplicity, we take $c = 1$. The important message here is that it is worth the effort to model non-uniform gain within the beamwidth. If we had assumed uniform gain to build our topologies we wouldn't have cared about the gain of communicating end nodes towards each other and, which as shown in Equations 4 and 5, makes a big difference in the power required for communication.

[1]It is usually measured in decibels (dB) and taken as 0 dB, which equals unity in absolute terms

[2]This follows naturally from the definition of beamwidth which is defined as angle between point of half gain on both sides of the axis of peak gain

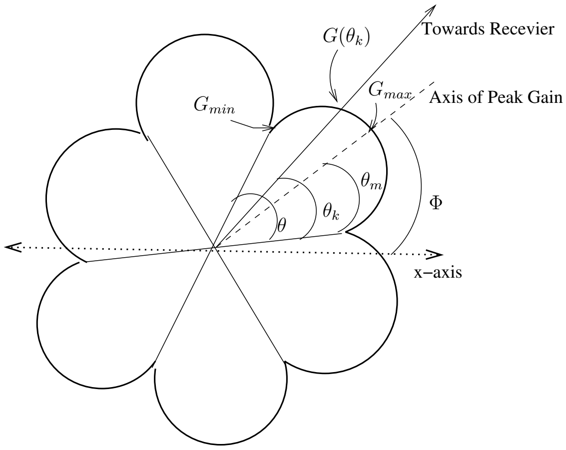

Fig. 1. Calculation of $G(\theta_k)$

2) Calculation of Gain: Since our intention is to model the radiation pattern of a switched beam antenna with non-uniform gain within the beamwidth, we need a function to calculate the gain associated with communication in different directions. The gain from a transmitter towards a receiver is a function of the angle of the straight line joining the two nodes and the beam margin. That is, in Figure 1, $G(\theta_k)$ is the gain of transmitting antenna towards a receiver k which is at an angle θ_k. Also suppose θ and θ_m are the beamwidth and the half-beamwidth respectively. Then $G(\theta_k)$ can be modelled by the following gaussian function:

$$G(\theta_k) \ = \ G_{max}.e^{-\frac{(\theta_k - \theta_m)^2}{2\sigma^2}} \qquad (6)$$

Here σ can be calculated by using the known value of $G(\theta_k)$ at $\theta_k = \theta$ giving $\sigma = \frac{\theta_m}{\sqrt{2ln2}}$

Thus, using Equation 6 each node can calculate the gain towards any direction. Combining this with Equation 1, the power required to reach another node can be calculated. More details about the above gaussian model can be found in [3], [6].

III. Problem Statement

Our algorithms require centralized execution and assume a static network. We formulate our problem of topology control with switched beam directional antennas into two cases.

Definition 1. *Connected MinMax Power with Given Antenna Orientation (CMPGA).* Given a set $M = (N, L, \Phi)$, with a set of B beams of beamwidth θ per node $\in N$, and path loss function γ, find a per node minimal assignment of power $p : N \rightarrow Z^+$ such that the induced graph is connected and $MAX_{u \in N, i \in B}(p(u, beam_i))$ is a minimum.

Definition 2. *Connected MinMax Power with Derived Antenna Orientation (CMPDA).* Given a set $M = (N, L)$, with a set of B beams of beamwidth θ per node $\in N$, and path loss function γ, find a antenna orientation assignment $\Phi : N \rightarrow \Phi, 0 \leq \Phi \leq \theta$ and a per node minimal assignment of power $p : N \rightarrow Z^+$ such that the induced graph is connected and $MAX_{u \in N, i \in B}(p(u, beam_i))$ is a minimum.

The CMPGA problem corresponds to settings where the antenna orientation is assumed given and not a parameter in topology construction. The solution to the CMPGA problem would be useful in settings where each node's antenna orientation cannot be changed after deployment. A pseudo-distributed algorithm where all nodes exchange their location and orientation information and find optimal maximum transmission powers themselves would find it useful. This problem however, may not be very relevant in actual settings where a centralized authority could control the antenna orientation at the deployment stage. Previous work assumed uniform gain within the beamwidth to build topologies and thus did not use antenna orientation as a parameter. Hence, we use the solution to the CMPGA problem also as a base case for comparison against the case where antenna orientation is a parameter in topology construction.

The CMPDA problem on the other hand deals with scenarios where the antenna orientations of each node can be calculated offline and deployed by a centralized authority or controlled by the node's user after deployment at the direction of an elected leader in the network. It is important to keep in mind that the communication system of a switched beam antenna cannot control the antenna orientation and needs external assistance. We assume the antenna is capable of being positioned with a discrete granularity of δ giving a total of $k = \frac{\theta}{\delta}$ steps of orientation in all.

The MinMax property pertains to minimizing the maximum power used by each node as opposed to that of the total network. The metric used supports the line of thought that a network's lifetime is maximized by trying to minimize each node's power consumption. Minimizing the total power consumption of the network might conserve some extra energy at some nodes, but this energy conserved at the expense of network disconnection is of little value.

For simplicity we assume that network connectivity can be achieved without exceeding the maximum power capabilities of individual nodes. Our algorithms can easily be adjusted for a certain maximum power limit for the radios of each node. We consider both DO and DD links and present algorithms to solve CMPGA and CMPDA problems for both cases. In all cases we seek a connected graph with all edge-relations among vertices symmetric. Any asymmetric edge relations are removed by post-processing. Symmetric links are a requirement for most routing and MAC protocols. For example, in the 802.11 MAC protocol the node that has a packet to send transmits a Request to Send (RTS) and awaits a Clear to Send (CTS) scheme from the next hop receiver. If the link were not symmetric, either the RTS or CTS would have needed to traverse over multiple hops requiring complex handling.

IV. Algorithms for CMPGA Problem

The algorithms for this case are optimal as we are given antenna orientations of each node and can construct the required topology using information of node locations.

A. Algorithm for DO Links

Algorithm 1 CMPGA-DO

Input: Network $M(N, L, \Phi)$, Gain function G and Path Loss exponent γ and a source node s

Output: Power levels $p(u, beam_i)$ for each $i \in B$ for each node $u \in N$ that induces a connected graph

1: Calculate G_u^v from each node u to all other nodes v using G and its antenna orientation $\Phi(u)$.
2: Initialize n clusters corresponding to each node and one semi-cluster SC at source node s
3: **while** number of clusters greater than one **do**
4: next edge to add $(u, v) = findNext()$
5: Add v to SC if $v \notin SC$
6: **if** adding edge (u, v) creates a bidirectional link between u and v **then**
7: $p(u, beam_u^v) = \frac{d_{u,v}^\gamma}{G_u^v}$
8: $p(v, beam_v^u) = \frac{d_{v,u}^\gamma}{G_v^u}$
9: Merge cluster(u) with cluster(v)
10: NumClusters = NumClusters - 1

Procedure(CMPGA-DO) $findNext()$

1: **for** each node $u \in SC$ **do**
2: **for** each node $v \in N - \{u\}$ **do**
3: **if** no edge from u to v and $cluster(u) \neq cluster(v)$ **then**
4: max-power-of-u-when-v-added[u,v] = *findPower(u,v)*
5: (u, v) = $min(\forall_{u,v}$ max-power-of-u-when-v-added(u,v))
6: return (u, v)

Procedure(CMPGA-DO) $findPower(u, v)$

1: $upower = \frac{d_{u,v}^\gamma}{G(u,v,\Phi(u))}$
2: return $upower$

The CMPGA-DO algorithm is used to solve the CMPGA problem for DO links. The algorithm begins by calculating the gains of nodes towards each other[3] using Equations $1, 5$ and 6. Information from L and Φ is used to find the distances and angles of nodes towards each other for the required calculations. A cluster is initialized at each node. A semi-cluster SC is initialized at source node s. The algorithm proceeds in a similar fashion to the prim's minimum spanning tree algorithm, but with adjustments required for the fact that the edges are directed and we seek a connected bi-directed graph. We add directed edges one by one and SC is used to keep track of which edges and vertices have been added so far. A cluster is a connected sub-graph with vertices having symmetric edge relations to each other with all asymmetric edge relations removed. Whenever an edge relation between a vertex pair becomes symmetric, we merge the clusters of these vertices. By merging two separate clusters, we reduce the number of clusters by one. A bi-directed cycle is never formed because if two vertices are already in the same cluster, an edge won't be added between them. Thus, if we started with n clusters, we will have a connected graph with symmetric edge relations among vertex pairs after the number of clusters is reduced to just one.

[3]The notation $beam_u^v$ denotes the beam of node u that faces towards node v (by joining a straight line from u to v).

Theorem 1. *Algorithm CMPGA-DO is an optimal solution for the CMPGA problem with DO links.*

Proof. Line 4 adds a directional edge between two nodes if they are in different clusters. Lines 2 and 3 ensure that if we end, the graph is connected and all nodes have been considered.

Let the notation $p(x, j)$ denote the maximum power used by a node x, say for one of its beams j. Assume the contrary that the maximum power returned by our algorithm is not the optimum. Consider a node u assigned the maximum power $p(u, j)$. This has happened, by line 7, when it was connecting to another node v not in its cluster. Now since we are considering new edges to be added to SC in non-decreasing order of power, there exists no path from u to v with symmetric edges where all nodes along the path have lesser power than $p(u, j)$.

$$\nexists path(u, v) : \forall x \in path(u, v), p(x) < p(u) \tag{7}$$

This is because if all the nodes on the path from u to v had lesser power, we would have had a path with symmetric edge relations and would have used that instead of trying to connect to v directly from u.
Let the maximum powers of all nodes from the optimum algorithm be $p_{opt}(i, j)$. By our contrary supposition, the maximum power of optimal algorithm, $OPT < p(u, j)$. Also, $\forall i(p_{opt}(i, j) \leq OPT < p(u, j)$ which gives us $p_{opt}(u, j) < p(u, j)$.
Now, if the above were to be true, it means that u is not connected directly to v and there was a path from u to v. i.e.

$$\exists path(u, v) : \forall x \in path(u, v), p(x) < p(u) \tag{8}$$

However, this is in contradiction to Equation 7. Hence, $p(u, j)$ is equal to OPT and our algorithm returns the optimal maximum power. □

B. Algorithm for DD Links

The CMPGA-DD algorithm solving CMPGA problem for DD links has a major difference with the DO version. The gains are calculated between nodes as before, but the power required now depends on gains of both end nodes of a link. Thus the power required is symmetrical and the algorithm does not have to deal with directional edges. Thus, we do not need to use a semi-cluster SC as in Algorithm 1 to keep track of edges added. A cluster is initialized for each node including the one for the source s, cluster(s). Now in each step we keep merging cluster(s) with the cluster of a node, say v, whose

Algorithm 2 CMPGA-DD

Input: Network $M(N, L, \Phi)$, Gain function G, Path Loss exponent γ and a source node s

Output: Power levels $p(u, beam_i)$ for each $i \in B$ for each node $u \in N$ that induces a connected graph

1: Calculate G_u^v from each node u to all other nodes v using G and its antenna orientation $\Phi(u)$.
2: Initialize n clusters corresponding to each node
3: Start from source node s
4: **while** Number of clusters greater than one **do**
5:   next node to add $v = findNext()$
6:   $p(u, beam_u^v) = \frac{d_{u,v}^\gamma}{G_u^v . G_v^u}$
7:   $p(v, beam_v^u) = \frac{d_{u,v}^\gamma}{G_v^u . G_u^v}$
8:   Merge cluster(s) and cluster(v)
9:   NumClusters = NumClusters - 1

Procedure(CMPGA-DD) $findNext()$

for each node $u \in cluster(s)$ **do**
  for each node $v \notin cluster(s)$ **do**
    max-power-of-u-when-v-added[u,v] = *findPower(u,v)*
(u, v) = $min(\forall_{u,v}$ max-power-of-u-when-v-added(u,v))
return (u, v)

addition through a node u of cluster(s), keeps the maximum power of network minimum. The algorithm terminates when the number of clusters is one with the same reasoning as Algorithm 1.

Theorem 2. *Algorithm CMPGA-DD is an optimal solution for the CMPGA problem with DD links*

Proof. Line 5 adds a node to the cluster of source. Lines 2 and 4 ensure that if we end, the graph is connected and all nodes have been considered.

Similar to the earlier proof for DO links let $p(u, j)$ denote the maximum power used by our algorithm and assume the contrary that there is an optimal algorithm which uses lesser power than $p(u, j)$. From line 5, this maximum power must have been used to directly connect to a node v not in the same cluster.

Let the maximum powers of all nodes from the optimum algorithm be $p_{opt}(i, j)$. By our contrary supposition, the maximum power of optimal algorithm, $OPT < p(u, j)$. Also, $\forall i(p_{opt}(i, j) \leq OPT < p(u, j)$ which gives us $p_{opt}(u, j) < p(u, j)$.

Now, if the above were to be true, it means that u is not connected directly to v and there was a path from u to v. i.e. v was added to u's cluster through some other node. This contradicts our earlier finding that $p(u, j)$ must have been used to directly connect to v. Hence, $p(u, j)$ must be the optimal maximum power of the network. □

Procedure (CMPGA-DD) $findPower(u, v)$

$upower = \frac{d_{u,v}^\gamma}{G(u,v,\Phi(u)).G(v,u,\Phi(v))}$
return $upower$

V. Algorithms for CMPDA Problem

We now present heuristic algorithms for the task of finding a power and orientation (p,Φ) assignment for all nodes such that the maximum power is minimized satisfying the connectivity constraint.

A. Algorithm for DO Links

Algorithm 3 CMPDA-DO

Input: Network $M(N, L)$, Gain function G, Path Loss exponent γ and a source node s

Output: Power levels $p(u, beam_i)$ for each $i \in B$ and $\Phi(u)$ for each node $u \in N$ that induces a connected graph

Initialize n clusters corresponding to each node and one semi-cluster SC at source node s
while number of clusters greater than one **do**
  next edge to add $(u, v) = findNext()$
  Add v to SC if $v \notin SC$
  if adding edge (u, v) creates a bidirectional link between u and v **then**
    $p(u, beam_u^v) = \frac{d_{u,v}^\gamma}{G_u^v}$
    $p(v, beam_v^u) = \frac{d_{v,u}^\gamma}{G_v^u}$
    Merge cluster(u) with cluster(v)
    NumClusters = NumClusters - 1

Procedure(CMPDA-DO) $findNext()$

for each node $u \in SC$ **do**
  for each node $v \in N - \{u\}$ **do**
    if no edge from u to v and $cluster(u) \neq cluster(v)$ **then**
      max-power-of-u-when-v-added[u,v] = *findMinPower(u,v,NbrList(u))*
(u, v) = $min(\forall_{u,v}$ max-power-of-u-when-v-added(u,v))
return (u, v)

Procedure(CMPDA-DO) $findMinPower(u, v, NbrList(u))$

for all steps of antenna orientation k **do**
  for each neighbor w of u including v **do**
    $upower_{k,w} = \frac{d_{u,w}^\gamma}{G(u,w,k)}$
  $maxpower[k]$ = $max(\forall_w upower_{k,w})$
return $min(\forall_k maxpower[k])$
Store optimal angle orientation k for adding v through u

The CMPDA-DO algorithm is similar to the CMPGA-DO algorithm. The main difference is in the way a new edge is selected to be added in SC based on the procedure *findNext*. The algorithm builds up its cluster from the source node s and adds a new edge to the SC based on which edge can

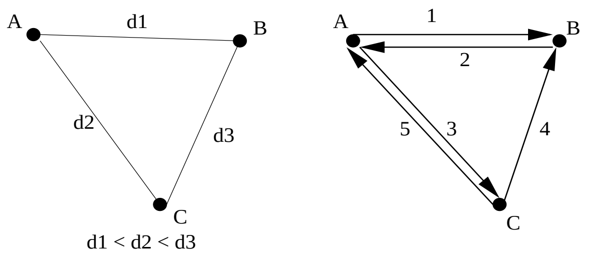

Fig. 2. CMPDA example showing step by step construction for a 3-node network

be added most cheaply in terms of power. Finding this 'best' edge involves checking all steps of antenna orientation for each node in SC for each stage, with the best orientation stored once a selection is made. The other point of note in this selection procedure is that the 'best' edge added to SC through a node $u \in SC$ is one that co-exists best (i.e minimizes maximum power) with the other already selected adjacent edges of u, $NbrList(u)$. Thus, when a new edge (u, v) is tried out by a node $u \in SC$, the maximum power it incurs for connecting to all nodes in $NbrList(u)$ and the new node v, is calculated for all possible antenna orientations of u. The orientation using minimum power among all these is stored as the best orientation for u to add v to the existing $NbrList(u)$. No antenna orientation information is used for gain calculations at the beginning of the algorithm, because the antenna orientation of nodes is varied as the algorithm proceeds to arrive at the final heuristic solution.

Figure 2 shows an example of the way the CMPDA-DO algorithm proceeds. Nodes A, B and C are separated by distances as shown. Node A, the source node, starts by adding B to the semi-cluster SC as B is the node that can be added with least power. This is because $d1$ is shortest and also because A can orient itself towards B providing maximum gain in that direction. The next cheapest link to be added is BA with a similar argument as above. Now both A and B form a cluster with bi-directed edges between them. The task now is to get a bi-directed edge from D to any of these two to form the required bi-directed connected graph. The algorithm checks k steps of orientation at A to decide what is best orientation at which A can add D (along with existing link to B) and keep its maximum power minimum. B does similarly. The node that will require minimum power among these adds the link to D. In this case A requires the minimum power to add D. D then adds a link to B as $d2$ is lesser than $d3$ and its antenna has no other links to care about. But since the network is still not bi-directed, D has to add a link to A before the algorithm terminates.

B. Algorithm for DD Links

The CMPDA-DD algorithm proceeds similarly as the CMPGA-DD algorithm but finds the next node to be added in a fashion similar to that of CMPDA-DO. In the procedure *FindMinPower*, one can notice that the same orientation step k is used on both ends of a DD link when finding the power

Algorithm 4 CMPDA-DD

Input: Network $M(N, L)$, Gain function G, Path Loss exponent γ and a source node s

Output: Power levels $p(u, beam_i)$ for each $i \in B$ and $\Phi(u)$ for each node $u \in N$ that induces a connected graph

1: Initialize n clusters corresponding to each node
2: Start from source node s
3: **while** Number of clusters greater than one **do**
4: next node to add $v = findNext()$
5: if G_u to any other adjacent node already in cluster(s) changes as a result of adding v, update the power required for those adjacent nodes of u
6: $p(u, beam_u^v) = \frac{d_{u,v}^\gamma}{G_u^v . G_v^u}$
7: $p(v, beam_v^u) = \frac{d_{v,u}^\gamma}{G_v^u . G_u^v}$
8: Merge cluster(s) and cluster(v)
9: NumClusters = NumClusters - 1

Procedure(CMPDA-DD) $findNext()$

1: **for** each node $u \in cluster(s)$ **do**
2: **for** each node $v \notin cluster(s)$ **do**
3: max-power-of-u-when-v-added[u,v] = *find-MinPower(u,v,NbrList(u))*
4: (u, v) = $min(\forall_{u,v}$ max-power-of-u-when-v-added(u,v))
5: return (u, v)

(see line 3). This is because, when a new node is to be added, it does not have any dependent adjacent nodes yet and hence can be adjusted to use any orientation. For $\theta \leq \frac{\pi}{2}$, which is the range of beamwidths we are interested in for this work, the angle of a node u towards another node v is the same in the other direction from v to u. One important thing to be taken care of is that when a new node v is added to cluster(s), it is likely that the node u in cluster(s), through which v was added, will change its orientation and thus gain. Since the links are DD, all adjacent nodes of u will have to re-adjust their powers to reach u and vice-versa (line 5 of Algorithm 4).

C. Discussion on accuracy and complexity of our heuristic algorithms

The maximum power required for topologies created by our heuristic algorithms in the worst case is no better than the algorithms of section IV. That is, the maximum powers could

Procedure(CMPDA-DD) $findMinPower(u, v, NbrList(u))$

1: **for** all steps of antenna orientation k **do**
2: **for** each neighbor w of u including v **do**
3: $upower_{k,w} = \frac{d_{u,w}^\gamma}{G(u,w,k).G(w,u,k)}$
4: $maxpower[k]$ = $max(\forall_w upower_{k,w})$
5: return $min(\forall_k maxpower[k])$
6: Store optimal angle orientation k for adding v through u

be as much as twice the optimal maximum power in case of DO links and upto four times as much for DD links. This happens because in our algorithm, once an edge is added, that edge is always kept. The next edge to be added is one which minimizes the maximum power of the node while retaining all its previously chosen edges and this new edge.

Definition 3. *The edges of distance d_1 and d_2 ($d_1 \geq d_2$) are said to be comparable if $\frac{d_1}{d_2} \leq (\frac{G_{max}}{G_{min}})^{1/\gamma}$ for DO links and $\frac{d_1}{d_2} \leq (\frac{G_{max}}{G_{min}})^{2/\gamma}$ for DD links.*

The definition of comparable edges comes intuitively from the range of influence of the gains. If an edge e_1 (of distance d_1) longer than an edge e_2 (of distance d_2) can be made to use lesser power than e_2 by assigning it G_{max} and assigning G_{min} to e_2, then the edges are comparable. The expressions in the definition come from assigning a gain of G_{max} and G_{min} to links with distances d_1 and d_2 respectively in Equation 1.

The key idea of our algorithm is that at each step it finds the best antenna orientation possible given the edges it has selected already. However, if two edges e_1 and e_2 have comparable distances, our algorithm can end up choosing the wrong one (say e_1) compared to the one an optimal solution(say e_2) may choose. That is because, at a later stage an edge e_3 may have to be added which goes better with e_2 than e_1. This is the approach we take to avoid the exponential complexity of checking all possibilities at all nodes. On the average case our heuristics are expected to do much better than the fixed orientation algorithms which is proved true by our evaluations in the following section. The complexity of our heuristic algorithms is $O(k.n^5)$ where k, as pointed out earlier, is the number of steps of antenna orientation. The more steps of antenna orientation we check, the better our solution at the cost of higher algorithm complexity.

VI. Evaluation

We aim to demonstrate through our evaluation the benefits of making the antenna orientation a parameter in topology construction. We show the effects of different beamwidth and δ on Maximum Transmit Power (MTP) for all four algorithms. In our experiments, the nodes were uniformly distributed within the area considered. Each data point shown is the average of 200 runs. The expression $G_{max} = \frac{2\pi}{\theta}$, modelling the gain of antenna along the azimuthal plane, was used to calculate the value of G_{max} for various values of beamwidth. Unless specified otherwise, we use a beamwidth of $30°$, δ equal to $1°$. For all experiments we vary the area size to vary node density keeping number of nodes constant at 50. The default area size is 100×100. For CMPGA experiments, each node is given an antenna orientation of $0°$ (refer Figure 1). The randomness in the locations of nodes is enough to provide randomness in gains of node pairs. 95% confidence intervals for the mean are shown where applicable.

A. Comparison of all four algorithms

We evaluated the MTP required for different node densities as shown in Figure 3. As expected, the MTP required increases with decreasing density with a big difference between transmit power required using DO and DD links. More important, however, is the reduction in MTP achieved by using the antenna orientation as a parameter in case of our heuristic algorithms. The reduction is not easy to make out in Figure 3 as the y-axis is represented in log-scale. So the corresponding reductions are shown in Figure 4. The power reductions are consistent across various densities with averages of about 17% and 19% for the DO and DD case.

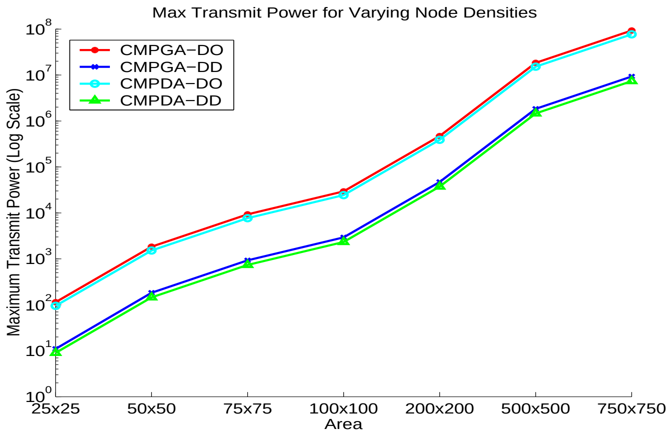

Fig. 3. Comparing MTP required by all schemes over different node densities

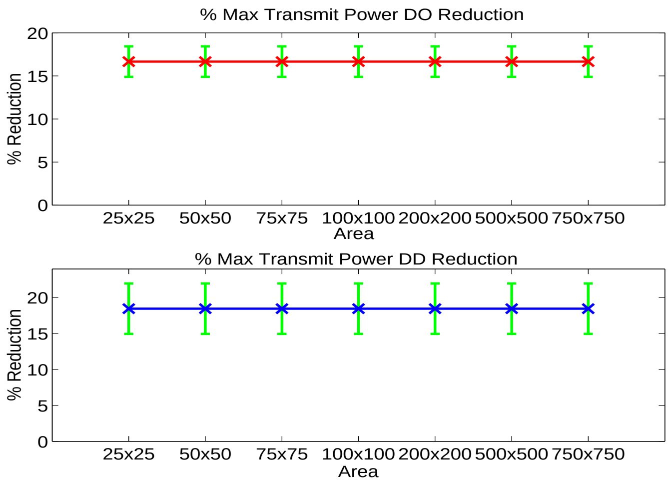

Fig. 4. Percentage savings by using antenna orientation as a parameter

B. Effect of varying beamwidth

We assume that when the beamwidth is decreased or increased, the number of beams increases or decreases, maintaining a $360°$ coverage around the node. Thus, increasing or decreasing the beamwidth does not have any positive or negative effects in terms of coverage. As Figure 5 shows, the effect of increased beamwidth is to require higher powers which follows from the fact that the value of maximum gain decreases with increasing beamwidth.

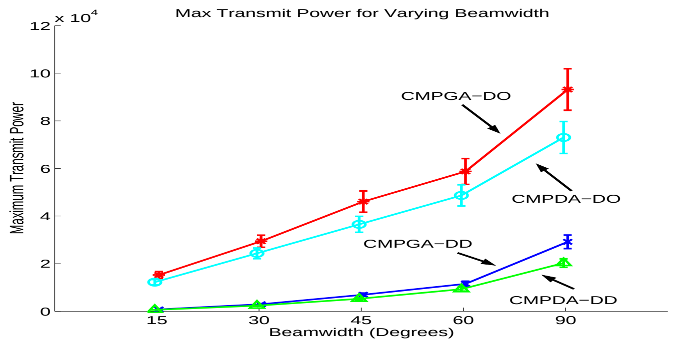

Fig. 5. Comparing the effect of beamwidth on maximum transmit power for all schemes

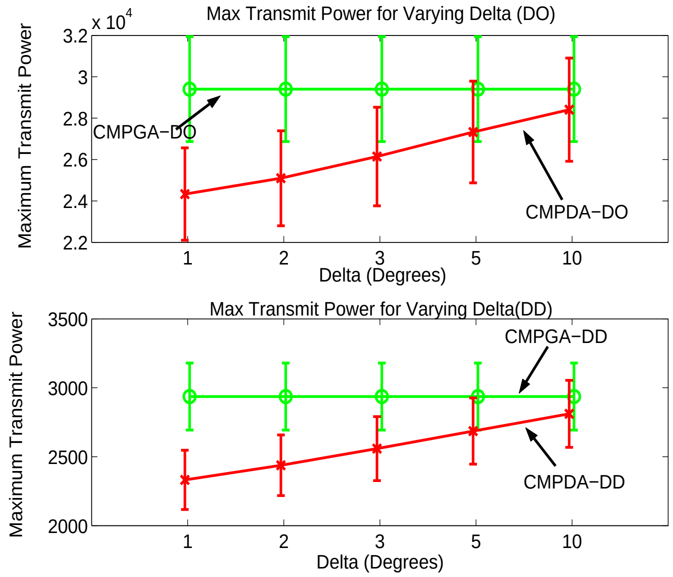

Fig. 6. Comparing the effect of delta on maximum transmit power

C. *Effect of varying δ*

The parameter δ is the granularity at which antenna orientation can be set. Smaller values of δ signify more control over finding a 'good' set of antenna orientations for all nodes leading to smaller values of MTP as shown in Figure 6. However, smaller values of δ also mean there are more steps of antenna orientation to be searched leading to increased complexity of the algorithms. Thus there is a tradeoff to be considered depending on various factors like how close to optimal values are required and the number of nodes in the network. Importantly, it can be noticed that even at higher values of δ, the CMPDA algorithms seem to do better than CMPGA.

VII. Conclusion and Future Work

We argued the case for modelling switched beam directional antennas with a non-uniform model of gain within the beamwidth. We presented optimal algorithms which construct a topology that minimizes the maximum power of the network while keeping it connected. We also presented heuristic algorithms which demonstrate that more power efficient topologies can be constructed if the antenna orientation is used as a parameter by the topology control algorithm. We studied the problem for both DO and DD links and found an average reduction in maximum transmit power of 17% and 19% for both cases respectively.

In future we intend to improve our heuristic algorithms to make them more efficient, and also adapt them for distributed settings. Optimizing for other cost metrics like total power of the network and achieving other network properties like bi-connectivity will also be addressed in our future research.

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